Control $\underline{\text{Sec 5}}$

* Frequency Domain Analysis

steady state Response to sinsoidal input

$$r(t) = A \sin(\omega t)$$

$$Y(s) = T(s)\, R(s)$$

$$\rightarrow R(s) = \frac{A}{s^2+\omega^2}$$

$$Y(s) = T(s)\, \frac{A}{s^2+\omega^2}$$

Assume :

$$T(s) = \frac{N(s)}{\prod_i (s+P_i)}$$

$$Y(s) = \frac{A\, N(s)}{(s^2+\omega^2) \prod_i (s+P_i)}$$

$$y(t) = \mathcal{L}^{-1}\left[Y(s) \right]$$

$$y(s) = \frac{C_1}{s+P_1} + \frac{C}{s+P_2} + \cdots\cdots + \frac{\alpha s+\beta}{s^2+\omega^2}$$

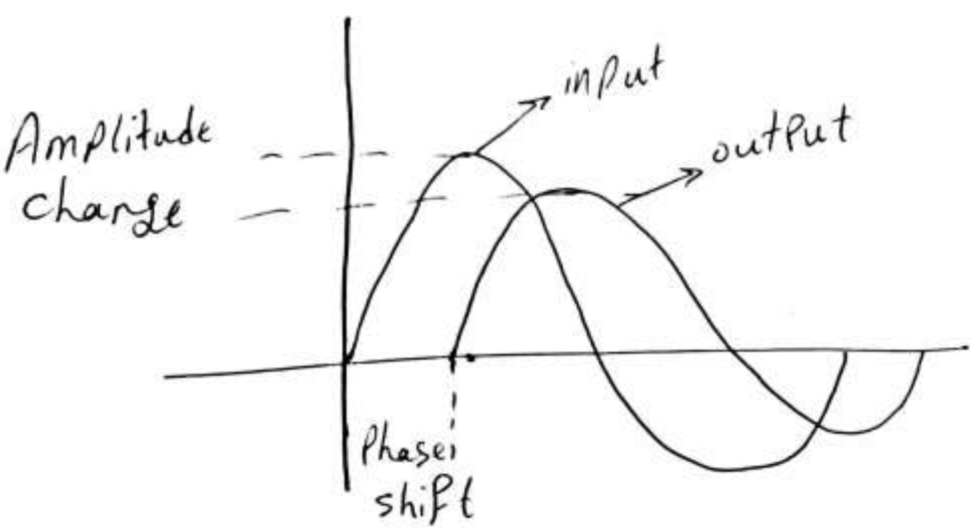
$$y(t) = C_1 e^{-P_1 t} + C_2 e^{-P_2 t} + \cdots\cdots + \alpha \cos(\omega t)$$

$$+ \frac{\beta}{\omega} \sin \omega t$$

$$y(t) = C_1 e^{-P_1 t} + C_2 e^{-P_2 t} + \cdots\cdots + C \sin(\omega t - \phi)$$

→ at steady state

$$y(t)\Big|_{t=\infty} = C \sin(\omega t - \phi)$$



input was $r(t) = A \sin(\omega t)$

→ Magnitude change & Phase shift ⇒ depend on frequency response of the system (Bandwidth)

* Fourier Transform

$$X(\omega) = \int_{-\infty}^{\infty} x(n)\, e^{-J\omega t} \cdot dt$$

From laplace Transform

$$X(s) = \int_{-\infty}^{\infty} x(n)\, e^{-st}\, dt$$

$s \rightarrow J\omega$ ⇒ Laplace → Fourier

For given system represented by T.F $T(s)$ replace of every s by $J\omega$ → The system will be described in frequency domain.

$$T.F : \underbrace{G \cdot H(J\omega)}_{\text{complex Function}} = \underbrace{|GH(J\omega)|}_{\text{Magnitude}} \underbrace{\angle GH(J\omega)}_{\text{phase shift}}$$

* Frequency domain Analysis

(1) Polar Plots.

(2) Bode diagram

"Bode diagram"

$$|GH(J\omega)| = \frac{\Pi_i |(J\omega + z_i)|}{\Pi_k (J\omega + P_k)}$$

$$\log|GH(J\omega)| = \log|(J\omega + Z_1)| + \log|(J\omega + Z_2)| + \cdots$$
$$-\log|(J\omega + P_1)| - \log|(J\omega + P_2)| + \cdots$$

$\rightarrow$ Bode diagram

$\rightarrow$ $\log|GH(J\omega)|$

$\Downarrow$

$\mathcal{L}_m$ v.s ω (log)

$\rightarrow$ ϕ v.s ω

$* \; s^{\pm n}$

$$\mathcal{L}_m = \pm 20\, n \,\mathcal{L}og(\omega)$$

سه هنرسم ال $\mathcal{L}_m$ v.s $\log(\omega)$ حتى ينتج خط مستقيم

ودة على ال (semi log)

$$\phi = \pm 90\, n$$

$* \; \underline{\underline{K \; s^{\pm n}}}$

$\to \mathcal{L}_m = 20 \log(K) \pm 20\, n \log(\omega)$

$\to \phi = \pm 90\, n$ <u>Cause</u> Phase of $K = 0$

$* \; (1 + Ts)^{\pm n}$

$$\omega \ll \frac{1}{T} \Rightarrow \mathcal{L}m = 0$$

$$\omega \gg \frac{1}{T} \Rightarrow \mathcal{L}m = \pm 20 \log(\omega T)$$

$$\phi = \pm n \tan^{-1}(\omega T)$$

$\boxed{Ex}$

$$GH(s) = \frac{K}{s(s+2)(s+10)}$$

K such that $e_{ss} \leqslant 0.1$ for unit ramp

$\boxed{sol}$

→ For unit ramp:

$$e_{ss} = \frac{1}{K_v} \quad , \quad K_v = \lim_{s \to 0} s\,GH(s)$$

$$K_v = \frac{1}{e_{ss}} = 10 \Rightarrow 10 = \lim_{s \to 0} \frac{sK}{s(s+2)(s+10)}$$

$$\therefore K = 200$$

$$\therefore GH(s) = \frac{200}{s(s+2)(s+10)}$$

① system → Time Constant form

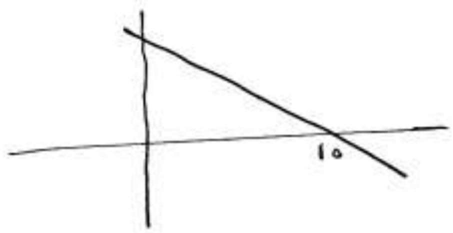
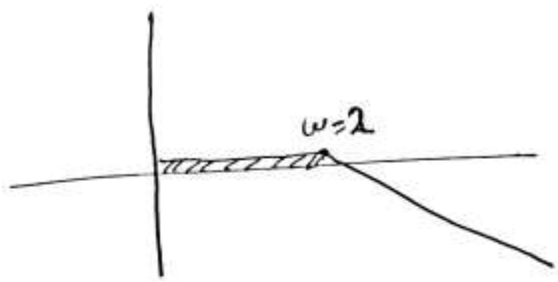
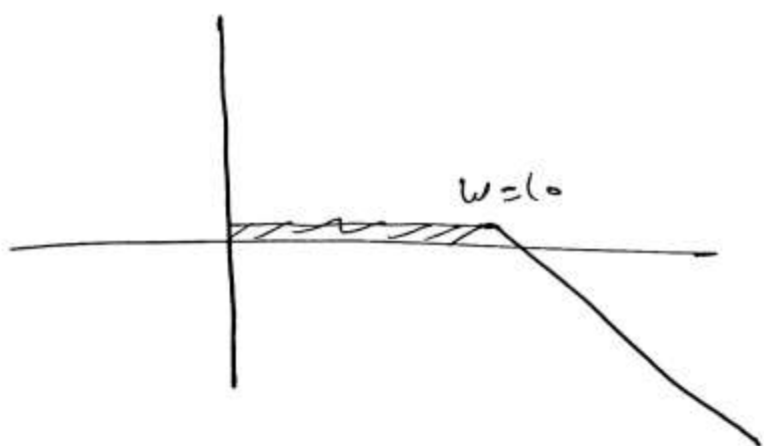
$$GH(s) = \frac{\cancel{200}^{\,10}}{10 * 2 * s(1+0.5s)(0.1s+1)}$$

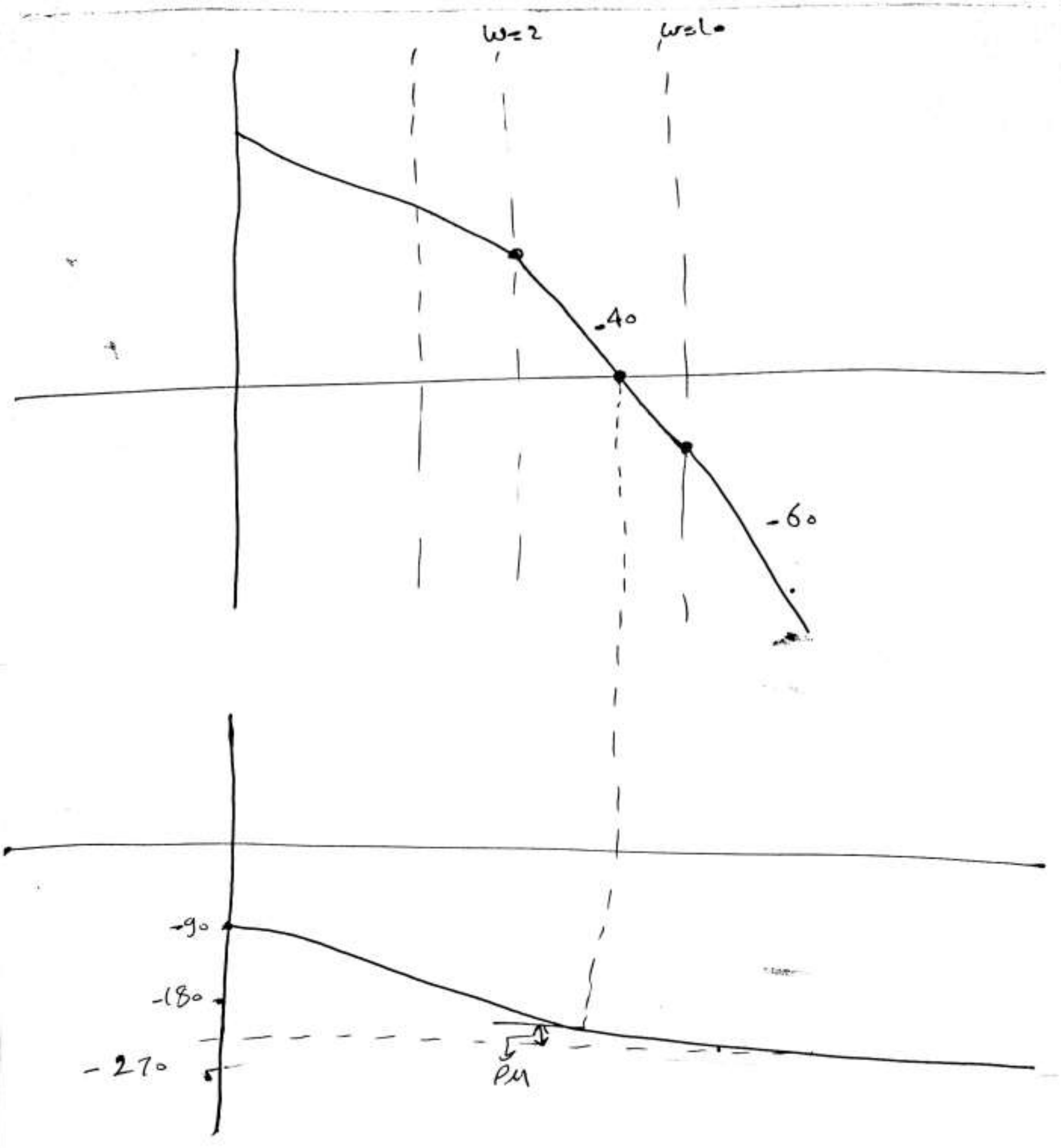
$\underline{s \longrightarrow j\omega}$

$$GH(j\omega) = \frac{10}{j\omega\,(0.5\,j\omega+1)\,(0.1\,j\omega+1)}$$

$\rightsquigarrow$ corner Frequency

$\omega_{c_1} = 2$, $\omega_{c_2} = 10$

term	Curve
$\frac{10}{j\omega}$	
$\frac{1}{j0.5\omega+1}$	
$\frac{1}{j0.1\omega+1}$	



$$\phi = -90 - \tan^{-1} 0.5 - \tan^{-1} 0.1\omega$$

ω	0	1	2	10	∞
ϕ	-90	-122.3	-146.3	-213.9	-270

$\boxed{8}$ secs

GM

$$GM = \frac{1}{M}\bigg|_{\phi=-180} \text{ at } \omega_{pc}$$

$$20\log GM = 20\log\frac{1}{M}$$

$$G.M|_{dB} = -20\log M = -L_m$$

$GM \to >1$ stable
$\to =1$ critically
$\to <1$ unstable

$GM|_{dB} \to >0$ stable
$\to =0$ critically
$\to <1$ unstable

$GM = 0 \quad \omega_{pc} = 4.5$

$$PM = 180 + \phi\big|_{M=1} \quad @\ \omega_{gc}$$

$$LM = 0$$

$\omega_{gc} = 4.5$

$PM = 0$